

F-theory and $N = 1$ Quivers from Polyvalent Geometry

Adil Belhaj¹ and Moulay Brahim Sedra²

¹Département de Physique, LIRST, Faculté Polydisciplinaire, Université Sultan Moulay Slimane

Béni Mellal, Morocco

² Département de Physique, LASIMO, Faculté des Sciences, Université Ibn Tofail, Kénitra, Morocco

Abstract

We study four dimensional quiver gauge models from F-theory compactified on four-folds with hyper-Kähler structure. Using intersecting complex toric surfaces, we derive a class of $N = 1$ quivers with charged fundamental matter placed on external nodes. The emphasis is on how local Calabi-Yau equations solve the corresponding physical constraints including the anomaly cancelation condition. Concretely, a linear chain of $SU(N)$ groups with flavor symmetries has been constructed using polyvalent toric geometry.

Keywords: *F-theory; D7-branes in Type IIB superstring; Quiver gauge theories; Toric geometry.*

1 Introduction

Study of supersymmetric gauge theories, in various dimensions, has attracted much attention. Concretely, they have been extensively studied in the context of II superstrings, M-theory, and F-theory using different technics. The main reason behind such an interest is that the relevant physics can be derived, nicely, from geometric and topologic data of the internal manifolds [1-7]. Very often in Calabi-Yau manifolds fibrations, the gauge group and matter content of the resulting models are associated with the singularities of the K3 surface fibers and the non-trivial base geometry respectively using the so-called *quiver method* [8]. The latter can be used to encode the field content of a gauge theory on a graph (quiver). Precisely, each node of the quiver is assigned a gauge group factor and to each link a matter, as the bi-fundamental and fundamental representations, charged under the local gauge symmetry. This approach has played a fundamental rôle in the interface of higher dimensional physics and geometry.

Recently, there have been many interesting developments in F-theory using geometric engineering method exploring either toric geometry or mirror symmetry, to deal with Calabi-Yau singularities. A special interest has been devoted to six-dimensional models from F-theory on an elliptically fibered Calabi-Yau threefold using the connection with ADE singularities based on Lie algebras [9]. It has been suggested that many four dimensional theories could be derived from a single six dimensional origin [10].

A close inspection, in the study of quivers, shows that the fundamental piece used in the deformation of the Calabi-Yau singularities is a collection of intersecting complex curves. Generally, they are identified with one complex dimensional projective spaces (\mathbf{CP}^1 's)[1, 2]. The corresponding intersection theory is intimately related to the Cartan matrices of Lie algebras. In this way, the Euler characteristic of toric manifold \mathbf{CP}^1 plays a more established rôle due to its connection with the diagonal entries of such matrices. In string theory, this connection has provided many quiver families with interesting physical properties. However, it is quite natural to think about quiver gauge theories based on other toric manifolds.

The purpose of this work is to contribute to such a geometric program by elaborating a class of $N = 1$ quivers in four dimensions obtained from intersecting two dimensional toric manifolds. This suggests a connection with F-theory compactification providing a new family of quivers that deserves a deep study. It may give a new meaning of the intersection of higher dimensional toric geometries encoding many physical information hidden in local Calabi-Yau manifold compactifications. More precisely, we discuss a class of quivers from F-theory compactified on four complex dimensional hyper-Kähler manifolds X^4 . The manifolds are defined as the moduli space of $N = 4$ sigma model in two dimensions. Analyzing the corresponding physical constraints, we reveal that these manifolds can be identified

with the cotangent fiber bundle over intersecting 2-dimensional complex toric varieties V^2 . Compactifying F-theory on these geometries, we obtain a class of $N = 1$ quivers based on intersecting V^2 's represented by polyvalent nodes. We show that the anomaly cancelation constraints can be solved using toric geometry and intersection theory of local Calabi-Yau manifolds.

This paper is organized as follows. Section 2 gives a concise review on F-theory. The compactification of F-theory on four complex dimensional hyper-Kähler manifolds X^4 is presented in section 3. The construction of the corresponding $N = 1$ quivers in four dimensions using toric geometry method is given in section 4. A possible generalization based on polyvalent geometry, producing a linear chain of $SU(N)$ groups with a flavor symmetry, is proposed in section 5. The last section is devoted to concluding remarks.

2 F-theory

In this section we give a quick review of the relevant geometric backgrounds of F-theory. Following Vafa [11], this theory describes a non-perturbative vacuum of Type IIB superstring in which the dilaton and axion moduli are considered as dynamical ones. This has been supported by the implementation of a complex scalar field interpreted as the complex structure parameter τ of an elliptic curve providing, with Type IIB superstring, a twelve-dimensional space-time. In this way, Type IIB superstring theory can be seen as the compactification of F-theory on the elliptic curve T^2 , by ignoring the size parameter contribution. Considering F-theory, one can build lower dimensional models by exploring the compactification on elliptically fibered Calabi-Yau manifolds[12]. The famous example playing a crucial rôle, in such activities, is F-theory on elliptically fibred K3 manifolds.

Geometrically, one build such fiber manifolds by varying the complex parameter τ over the dimensional projective space \mathbf{CP}^1 parameterized by the local coordinate z . In this way, τ becomes a function of z as it varies over the \mathbf{CP}^1 base of such elliptic K3 surfaces. This class of manifolds can be obtained by considering complex surfaces given by the following equations

$$y^2 = x^3 + f(z)x + g(z) \quad (1)$$

where $f(z)$ and $g(z)$ are polynomials of degree 8 and 12, respectively. The homogeneity condition is required by the triviality of the canonical line bundle ensuring the Calabi-Yau condition. This equation has been extensively studied in many places in connection with elliptic fibration of F-theory [6, 7]. It is shown that these manifolds have 24 singular points associated with $\tau(z) \rightarrow \infty$. These singular points are associated with the vanishing condition of the following discriminant

$$\Delta = 4f^3(z)x + 27g^2(z). \quad (2)$$

In D-brane physics, these singularities have been identified with D7-brane locations producing interesting gauge theories in eight dimensions[11, 12]. However, the connection with real world requires four complex dimensional manifolds in the compactification scenario. In fact, one can consider F-theory on elliptic K3 fibration over a complex surface S . Indeed, $N = 1$ supersymmetry in four dimensions can be obtained by imposing some geometric constraints on S . It has been shown that S should satisfy the following Hodge number condition

$$h^{1,0} = h^{2,0} = 0. \quad (3)$$

In [6, 7], it has been revealed that these two conditions kill the adjoint chiral fields associated with Wilson lines and the zero mode of the canonical line bundle producing models with only four charges in four dimensions.

In what follows, such surfaces will be identified with two dimensional toric manifolds, which will be noted by V^2 . The corresponding intersection theory will be explored to build a class of $N = 1$ quivers with flavor symmetries.

3 F-theory geometric backgrounds

In this section, we discuss F-theory on four complex dimensional manifolds X^4 associated with the following compactification

$$\mathbb{R}^{1,3} \times X^4. \quad (4)$$

Mimicking the analysis used on the geometric engender of $N = 2$, we consider a class of manifolds satisfying the physical condition given in eq.(3). The manifolds will be identified with local backgrounds considered as the cotangent fiber bundle over intersecting two dimensional toric varieties V^2 . This may offer as a possible generalization of intersecting one dimensional projective spaces \mathbf{CP}^1 's appearing in local K3 surfaces. It is recalled that V^2 can be represented by toric diagrams (polytopes) $\Delta(V_2)$ spanned $2 + r$ vertices v_i belong to the Z^2 lattice [13, 14, 15]. These vertices verify toric constraints given by

$$\sum_{i=1}^{2+r} q_i^a v_i = 0, \quad a = 1, \dots, r, \quad (5)$$

where q_i^a are the corresponding Mori vectors. Probably, the most simple example is \mathbf{CP}^2 defined by $r = 1$ and the Mori vector charge $q_i = (1, 1, 1)$. The corresponding polytope has 3

vertices v_1, v_2 and v_3 of the Z^2 square lattice satisfying the toric relation

$$v_1 + v_2 + v_3 = 0. \quad (6)$$

This defines a triangle $(v_1 v_2 v_3)$, described by the intersection of three \mathbf{CP}^1 curves, in the R^2 plane.

Roughly speaking, F-theory local geometries that we will examine here can be constructed using technics of $N = 4$ sigma model in two dimensions [16, 17, 18]. Precisely, they are interpreted as the moduli space of two dimensional $N = 4$ supersymmetric field theory with $U(1)^r$ and $(r + 2)$ hyper-multiplets assuring the right dimension on which F-theory should be compactified. In this way, the manifolds are obtained by solving the following D-flatness conditions

$$\sum_{i=1}^{r+2} q_i^a [\phi_i^\alpha \bar{\phi}_{i\beta} + \phi_i^\beta \bar{\phi}_{i\alpha}] = \vec{\xi}_a \vec{\sigma}_\beta^\alpha, \quad a = 1, \dots, r \quad (7)$$

Here, q_i^a describe now the sigma model matrix charges. For each hypermultiplet, $\phi_i^{\alpha'}$ s ($\alpha = 1, 2$) are the component field doublets. It is noted that $\vec{\xi}_a$ are interpreted as the Fayet-Iliopolos (FI) 3-vector couplings related by $SU(2)$ symmetry. $\vec{\sigma}_\beta^\alpha$ are the traceless 2×2 Pauli matrices. It is remarked that the solutions of eqs(7) depend on many physical data including the number of the gauge fields, charges and the values of the FI couplings. Using $SU(2)$ R-symmetry transformations $\phi^\alpha = \varepsilon^{\alpha\beta} \phi_\beta$, $\bar{\phi}^\alpha = \bar{\phi}_{\alpha'}$, $\varepsilon_{12} = \varepsilon^{21} = 1$ and exploring the Pauli representation, eqs.(7) can be split as follows

$$\begin{aligned} \sum_{i=1}^{r+2} q_i^a (|\phi_i^1|^2 - |\phi_i^2|^2) &= \xi_a^3 \\ \sum_{i=1}^{r+2} q_i^a \phi_i^1 \bar{\phi}_i^2 &= \xi_a^1 + i \xi_a^2 \\ \sum_{i=1}^{r+2} q_i^a \phi_i^2 \bar{\phi}_i^1 &= \xi_a^1 - i \xi_a^2. \end{aligned} \quad (8)$$

Since eqs.(8) is invariant under the $U(1)^r$ gauge transformations, we can deduce precisely an eight-dimensional toric hyper-Kähler manifolds, which will be considered as F-theory backgrounds. However, explicit solutions of these geometries depend on the values of the FI couplings. Taking $\xi_a^1 = \xi_a^2 = 0$ and $\xi_a^3 > 0$, eqs.(8) describe the cotangent fiber bundle over complex two-dimensional toric varieties. To understand these solutions in some detail, we consider the case associated with the cotangent fiber bundle $T^*(\mathbf{CP}^2)$. The manifold is defined as the moduli space of $2D$ $N = 4$ supersymmetric $U(1)$ gauge theory with one isotriplet FI coupling parameter $\vec{\xi} = (\xi^1, \xi^2, \xi^3)$ and only three hypermultiplets of charges

$q_a^i = q^i = 1$, where $i = 1, 2, 3$. In this case, eqs.(8) can be reduced be

$$\begin{aligned}\sum_{i=1}^3 (|\phi_i^1|^2 - |\phi_i^2|^2) &= \zeta^3 \\ \sum_{i=1}^3 \phi_i^1 \bar{\phi}_{i2} &= \zeta^1 + i\zeta^2 \\ \sum_{i=1}^3 \phi_i^2 \bar{\phi}_{i1} &= \zeta^1 - i\zeta^2.\end{aligned}\tag{9}$$

It is noted that, for $\zeta^1 = \zeta^2 = \zeta^3 = 0$, the moduli space has an $SU(3) \times SU(2)_R$ symmetry. It can be considered as a cone over a seven manifold given by

$$\sum_{i=1}^3 (\varphi_{\alpha i} \bar{\varphi}_i^\beta - \varphi_i^\beta \bar{\varphi}_{\alpha i}) = \delta_\alpha^\beta.\tag{10}$$

However, the case $\vec{\zeta} \neq \vec{0}$ shows tat the $SU(3) \times SU(2)_R$ symmetry is explicitly broken down to $SU(3) \times U(1)_R$. When $\zeta^1 = \zeta^2 = 0$ and ζ^3 positive definite, we recover the cotangent fiber bundle over \mathbf{CP}^2 . For $\phi_i^2 = 0$, one finds

$$|\phi_1^1|^2 + |\phi_2^1|^2 + |\phi_3^1|^2 = \zeta^3,\tag{11}$$

defining now the \mathbf{CP}^2 projective space, in $N = 2$ sigma model language [19]. On the other hand, with $\zeta^1 = \zeta^2 = 0$ conditions, the two last equations of (9) mean that $\bar{\phi}_{2i}$ lies in the cotangent space to \mathbf{CP}^2 . This can be viewed as an extension of the canonical line bundle over \mathbf{CP}^2 used in the study of $\mathcal{N} = 1$ quivers embedded in type II superstrings. Putting $\zeta_i^2 = \zeta_i^3 = 0$ and introducing $x_i = \phi_i^1$ and $y_i = \phi_i^2$, the two last equations (8) can be rewritten as

$$\begin{aligned}\sum_{i=1}^3 x_i \bar{y}_i &= 0 \\ \sum_{i=1}^3 \bar{x}_i y_i &= 0,\end{aligned}\tag{12}$$

interpreted as two orthogonal variables. Then, the total space is a cotangent fiber bundle over \mathbf{CP}^2 . Locally, it can be identified with

$$C^2 \times V^2\tag{13}$$

To connect this geometry with the elliptic curve fibration, we orbifold the fiber C^2 by a by subgroups Γ of $SU(2)$ to build spaces of the form \mathbf{C}^2/Γ . This can produce a fibration of an

elliptic curve over the plan

$$\mathbf{C}/\Gamma \times \mathbf{C} \quad (14)$$

In fact, the elliptic curve \mathbf{C}/Γ can be controlled by eq.(1). This can produce local K3 surfaces associated with D7-brane locations in Type IIB superstring. The construction of such manifolds goes back to many years ago and it has been explored to elaborate *ADE* gauge symmetries in eight dimensions [11, 12].

Motivated by Standard Model (SM) and its extensions, it should be interesting to consider four dimensional quiver gauge theories from such geometries. In fact, this can be obtained by considering intersecting two dimensional complex toric V^2 's. Indeed, the previous example can be generalized by considering $U(1)^r$ gauge theory in $N = 4$ sigma model. This can be analyzed to show that eqs.(7) describe the cotangent fiber bundle over r intersecting V_a^2 ($a = 1, \dots, r$). These geometries can arise as a generalization of intersecting \mathbf{CP}^1 's of the K3 surfaces which are classified by Lie algebras [11, 12].

In the present geometry, V_a^2 form a basis of the middle cohomology group $H_4(X^4, \mathbb{Z})$. The corresponding intersection numbers are given by the square matrix

$$[V_a^2] \cdot [V_b^2] = I_{ab}. \quad (15)$$

Toric geometry assures that V_a^2 intersects V_{a+1}^2 at a single \mathbf{CP}^1 which produces the following intersection numbers

$$[V_a^2] \cdot [V_{a+1}^2] = -2. \quad (16)$$

Assuming that V_a^2 does not intersect V_b^2 if $|b - a| > 1$. As in the case of \mathbf{CP}^1 's, the intersection matrix should take the following form

$$I_{ab} = n\delta_{ab} - 2(\delta_{a, b-1} + \delta_{a, b+1}), \quad (17)$$

where n is the Euler characteristic of V^2 , noted usually by $\chi(V^2)$.

Having constructed a special type of fourfolds manifolds. The next section will be concerned with the associated quiver. Concretely, we will discuss the connection between the physics content of F-theory on such manifolds and quiver method of $N = ?$ supersymmetric gauge theories in four dimensions. The analysis that will be used here is based on polyvalent toric geometry.

4 Quiver gauge theories from F-theory

To start, it is recalled that four dimensional gauge theories has attracted a special attention in connection with string theory on Calabi-Yau manifolds. Concretely, they have been investi-

gated in the study of D-brane gauge theories obtained from the compactification on singular Calabi-Yau manifolds or G_2 manifolds [4]. In this way, the gauge symmetry and the matter content of the resulting models can be derived from the geometric and topological data of the internal manifolds[1-7]. The associated physical parameters are related to their moduli spaces. A nice way to encode the physical information of such gauge theories is to use the quiver method[8]. The corresponding field models are usually referred to quiver gauge theories. In this way, The physical content of a model with several gauge group factors

$$G = \prod_a G_a, \quad (18)$$

can be illustrated by a quiver. As in graph theory, it is formed by nodes and links. For each node, one associates a gauge factor G_a , where a indicates the number of nodes. Here, it can be identified with the number of V_a . However, links between two nodes are associated with charged matter transforming either in bi-fundamental or fundamental representations of the gauge group G .

In string theory, many examples of such quivers have been elaborated in connection with toric graphs and Dynkin diagrams. It is worth noting that the last class is the most studied one due to its link with D-brane gauge theories obtained from Calabi-Yau singularities classified by Cartan matrices of Lie algebras. This has led to a nice classification of stringy quivers. Details on the corresponding materials can be found in, for example, [1, 2]. It is remarked that these classes have been based on the intersection of \mathbf{CP}^1 's.

Using higher dimensional intersecting geometries, we would like to engineer $N = 1$ quivers in the F-theory contexts using Type IIB superstring dual descriptions in terms of D7-branes. It is recalled that F-theory contains also D3 and D5-branes localized at some singular points. However, to understand the present construction, we focus for simplicity, on D7-branes wrapping two complex dimensional toric varieties and filling the four-dimensional Minkowski space. Roughly speaking, it has been shown that F-theory near the A_{N-1} singularity of the K3 surfaces is equivalent to N units of D7-branes in Type IIB dual producing $SU(N)$ gauge theories in eight dimensions [11, 12]. Four dimensional quiver gauge theories can be reached by wrapping such D7-branes on intersecting V_a^2 . Indeed, consider r different stacks of D7-branes. By standard results in quiver gauge theories, the associated gauge group reads as

$$G = \bigotimes_{a=1}^r SU(N_a), \quad N = \sum_a N_a \quad (19)$$

where N_a are integers which characterize the physical content. In general, these integers are determined by imposing physical requirements on the internal manifolds. In the presence of fundamental matter, the anomaly cancelation condition for the theory with only four

supercharges responds to the following constraint equations

$$\sum_{a=1}^r I_{ab} N_a - M_b = 0, \quad (20)$$

where I_{ab} is the intersection matrix of V^2 's on which D7-branes are wrapped. M_b are fundamental matter fields, charged under the gauge group G . To get the corresponding quivers, one should solve these physical constraints. A priori, there are many ways to solve them. However, a close inspection shows that it can be interpreted as a toric geometry realization of local Calabi-Yau manifolds [2]. To understand this way in some details, we present explicit models. For example, we consider two gauge factors $U(N_1) \times U(N_2)$ quiver gauge theory associated with two intersecting generic toric manifold V^2 's at \mathbf{CP}^1 . The main simplifying feature of this geometry is that its intersection matrix reads as

$$I_{ab} = \begin{pmatrix} n & -2 \\ -2 & n \end{pmatrix}. \quad (21)$$

To solve the associated physical equations (20), one may implement auxiliary nodes producing a new quiver with more than two gauge factors. We note that this geometric procedure allows one to modify the above intersection matrix leading to a Calabi-Yau geometry without affecting the dynamical gauge factors. Indeed, to get the desired quiver the extra gauge factors behave like non dynamic ones and they should be associated with fundamental matter fields. In geometric engineering method, this can be obtained by assuming that auxiliary nodes correspond to D7-branes wrapping in non compact cycles. Geometrically, one should consider the following modified intersection matrix

$$\tilde{I}_{ai} = \begin{pmatrix} 2-n & n & -2 & 0 \\ 0 & -2 & n & 2-n \end{pmatrix} \quad (22)$$

as required by the local Calabi-Yau condition

$$\sum_{i=1}^4 \tilde{I}_{ai} = 0, \quad a = 1, 2. \quad (23)$$

The associated toric geometry is defined by a polyhedron which is a convex hull of four vertices

$$v_i = (v_i^1, v_i^2, v_i^3), \quad i = 1, \dots, 4 \quad (24)$$

satisfying the following toric relations

$$\sum_{i=1}^4 \tilde{I}_{ai} v_i = 0, \quad a = 1, 2. \quad (25)$$

In fact, these equations look like a generalization of (20). Up to some details, the corresponding physical quantities, like the number of fundamental matter and the gauge group ranks, can be identified with toric data including the Mori vector charges. One can obviously see that one of the vertex entries should satisfy the following requirement

$$\begin{aligned} v_1^1 &= M_1 \\ v_2^1 &= N_1 \\ v_3^1 &= N_2 \\ v_4^1 &= M_2 \end{aligned} \quad (26)$$

In turn, the intersection matrix can be regarded as Mori vectors and take the form

$$I_{ab} = \tilde{I}_{ab}, \quad a = 1, 2. \quad (27)$$

It follows from toric geometry that the equation (20) can be solved by

$$\begin{aligned} N_1 &= N_2 = N \\ M_1 &= M_2 = (n - 2)N. \end{aligned} \quad (28)$$

This solution can produce a quiver with fundamental charged matter which can be obtained by zero limits of the gauge coupling constants associated with two auxiliary nodes. In toric geometry, they correspond to the extra lines and columns of the triangular matrix (22). In practice, this can be done by considering cycles with very large volume. In this way, the dynamics associated with such cycles become weak and lead to a spectator flavor symmetry. This assumption provides a quiver theory with $U(N) \times U(N)$ gauge symmetry and flavor group of type $U(nN - 2N) \times U(nN - 2N)$ placed non physical nodes. The quiver illustrating the physics content is shown in figure 1. In this quiver, external nodes represent gauge symmetry factors, while the internal ones indicate flavor factors.

It is interesting to note this model is based on bivalent geometry. The analysis may be extended to quivers involving nodes with more than two links. This introduces polyvalent vertices in toric geometry representation of local Calabi-Yau manifolds. We discuss this issue in the following sections.

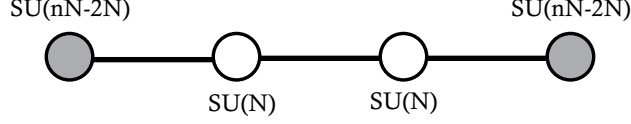


Figure 1: Bivalent quiver.

5 Quivers from trivalent geometry

In the engineering method used in Type II superstrings, trivalent geometry however contains both bivalent and trivalent nodes. It contains central nodes linked with three other ones. In the present F-theory backgrounds, a central node will correspond to a Mori vector charge of the form

$$(n, -2, -2, 4 - n, 0, \dots, 0), \quad (29)$$

required by the local Calabi-Yau condition. To get the quiver with such a geometry, one may follow the approach used for the bivalent one. Indeed, we consider the geometry associated with $U(1)^r$ gauge symmetry in $N = 4$ sigma model. It is observed that the trivalent geometry appears for $r \geq 3$. To see how this works, we first deal with a concrete example for $r = 3$. The local Calabi-Yau condition imposes the following modified intersection matrix

$$\tilde{I}_{ai} = \begin{pmatrix} 2-n & n & -2 & 0 & 0 & 0 \\ 0 & -2 & n & -2 & 4-n & 0 \\ 0 & 0 & -2 & n & 0 & 2-n \end{pmatrix}. \quad (30)$$

In order to construct the corresponding quiver, the equation (20) should be solved. As the previous model, the relevant physical data can be easily computed using toric geometry. We, then, find

$$\begin{aligned} N_1 &= N_2 = N_3 = N \\ M_1 &= M_3 = (n-2)N, \quad M_2 = (n-4)N. \end{aligned} \quad (31)$$

This solution indicates that one expects a quiver theory with $U(N)^3$ gauge symmetry and a flavor group of type $U(nN-2N)^2 \times U(nN-4N)$ associated with non physical nodes. This can be represented by a graph as shown in figure 2.

The general cases can be treated very similarly. It is easy to extend the above matrices to

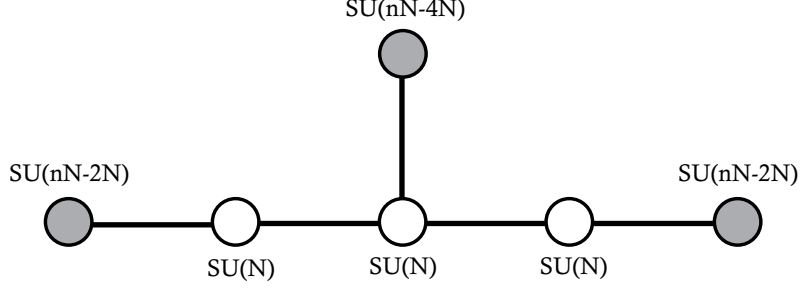


Figure 2: Trivalent quiver.

a class of models controlled by the following modified intersection matrix

$$\begin{aligned}
\tilde{I}_{00} &= \tilde{I}_{r+1r+1} = (2-n), \\
\tilde{I}_{ab} &= I_{ab}, \quad a = 1, \dots, r \\
\tilde{I}_{aa*2} &= (4-n), \quad a = 1, \dots, r-1
\end{aligned} \tag{32}$$

with other vanishing. Using toric geometry, it is possible to solve the equation (20). As the previous cases, the associated quiver can be obtained by zero limits of the gauge coupling constants corresponding to two bivalent and $r-2$ trivalent external nodes. This can produce a quiver theory with a $U(N)^r$ gauge symmetry and flavor group of type $U(nN-4N)^{r-2} \times U(nN-2N)^2$ associated with the spectator nodes. In graph theory, this model can be represented by the quiver given in figure 3.

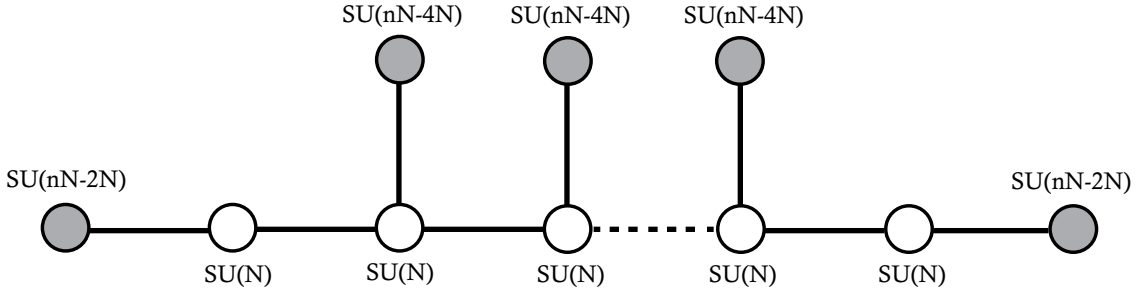


Figure 3: Extended trivalent quiver.

6 Polyvalent geometry and quivers

In this section, we construct quivers involving polyvalent nodes where the bivalent and trivalent ones appear just as the leading objects of a more general picture. This geometry

can be considered as a possible extension to the previous cases dealing with bivalent and trivalent quivers. It is recalled that a polyvalent node can represent a central toric variety V^2 with self intersection n intersecting m other ones with contribution (-2) . Like for the bivalent and trivalent case, the local Calabi-Yu condition impose the implementation of an extra non compact toric variety with contribution $2m - n$. This requires that F-theory quiver geometry should involve the following Mori vector

$$(n, \underbrace{-2, -2, \dots, -2}_m, 2m - n, 0, \dots, 0). \quad (33)$$

Instead of being general, let us consider a concrete example. Probably, the most simple one is a quiver involving just one tetravalent node. In geometric engineering method, this has been considered as is a particular example of polyvalent nodes coming after the bivalent and trivalent ones. In connection with Dynkin diagrams, the tetravalent geometry appears in the affine $\text{so}(8)$ Dynkin diagram. To understand that, we illustrate the tetravalent geometry. The quiver corresponds to 4 intersecting generic toric manifold V^2 's at \mathbf{CP}^1 controlled by the following matrix

$$I_{ab} = \begin{pmatrix} n & -2 & 0 & 0 \\ -2 & n & -2 & -2 \\ 0 & -2 & n & 0 \\ 0 & -2 & 0 & n \end{pmatrix}. \quad (34)$$

Local Calabi-Yau condition (20) requires the introduction of an auxiliary node producing a tetravalent geometry. The latter correspond to the following Mori vector

$$(n, -2, -2, -2, 6 - n, 0, 0, 0). \quad (35)$$

To solve the previous physical conditions, toric geometry produces the following equations

$$\sum_{i=1}^8 \tilde{I}_{ai} v_i = 0, \quad a = 1, 2, 3, 4 \quad (36)$$

with the local Calabi-Yau condition

$$\sum_{i=1}^8 \tilde{I}_{ai} = 0, \quad a = 1, 2, 3, 4 \quad (37)$$

Similar calculations can be used to build a quiver with fundamental charged matter placed on a tetravalent and 3 bivalent nodes as shown in figure 4.

This geometry provides a quiver theory with $U(N)^4$ gauge symmetry and flavor group of

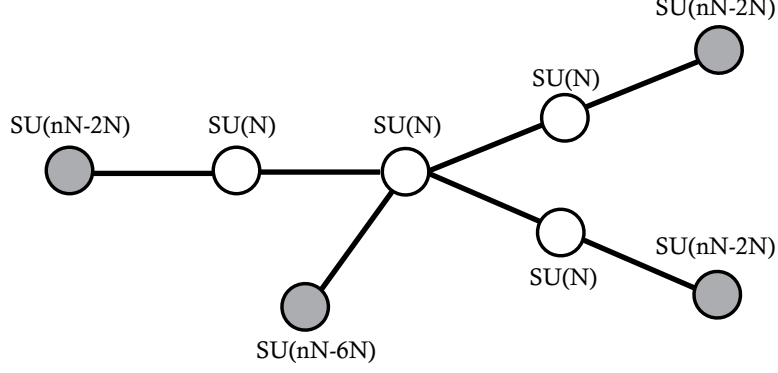


Figure 4: Tetravalent quiver.

type $U(nN - 2N)^3 \times U(nN - 6N)$.

7 Discussions

In this work, we have investigated a class of $N = 1$ quivers in four dimensions obtained from intersecting two dimensional toric manifolds. This suggests a connection with F-theory compactification providing a new family of quivers that deserves a deep study. It may give a new meaning of the intersection of higher dimensional geometries encoding many physical information hidden in local Calabi-Yau manifold compactifications. Concretely, we have engineered a class of quivers from F-theory compactified on four complex dimensional hyper-Kähler manifolds X^4 . The manifolds are defined as the moduli space of $N = 4$ sigma model in two dimensions. In particular, we have remarked that these manifolds can be identified with the cotangent fiber bundle over intersecting 2-dimensional complex toric varieties V^2 . Compactifying F-theory on these geometries, we have derived a class of $N = 1$ quivers based on intersecting $V^{2'}$ s. Then, we have shown that the anomaly cancelation constraints can be solved using toric geometry of local Calabi-Yau manifolds.

Explicitly, we have engineered quivers based on a linear chain of $SU(N)$ groups with flavor symmetries depending on polyvalent nodes. It turns out that each node a represents a toric variety $V_{a,m}^2$ which intersects m other ones. This node encapsulates a $SU(N_c)$ gauge factor and $SU(N_f)$ flavor symmetry. Calculation, in toric geometry, shows that

$$\begin{aligned} N_c &= N \\ M_f &= (n - 2m - 2)N. \end{aligned} \tag{38}$$

This work comes up with many open questions. Motivated by [20], it should be interesting to build the metric of such varieties. On the other hand, it has been observed that there is

a similarity with quivers based on finite Lie algebras. The approach could be adaptable to a broad variety of geometries associated with other Lie algebras. It would therefore be of interest to try to extract physical information on such geometries to give a complete study.

Acknowledgments: The authors thank Z. Benslimane for associated collaborations.

References

- [1] S. Katz, A. Klemm, C. Vafa, *Geometric engineering of quantum field theories*, Nucl. Phys. **B 497** (1997) 173, hep-th/9609239.
- [2] S. Katz, P. Mayr, C. Vafa, *Mirror symmetry and exact solution of 4d $N = 2$ gauge theories I*, Adv. Theor. Math. Phys. **1** (1998) 53, hep-th/9706110.
- [3] J. J. Heckman, C. Vafa, *From F-theory GUTs to the LHC*, 0809.3452[hep-th].
- [4] B. S. Acharya, K. Bobkov, *Kahler Independence of the G2-MSSM*, arXiv:0810.3285[hep-th].
- [5] A. Belhaj, L. J. Boya, A. Segui, *Holonomy Groups Coming From F-Theory Compactification*, Int.j.Theor.Phys 49(2010)681, arXiv:0911.2125.
- [6] C. Vafa, *On $N = 1$ Yang-Mills in four dimensions*, Adv. Theor. Math. Phys. **2** (1998) 497, hep-th/9801139.
- [7] S. Katz, C. Vafa, *Matter From Geometry*, Nucl. Phys. **B497**(1997)146, hep-th/9606086.
- [8] M. Douglas and G. Moore, *D-Branes, Quivers, and ALE Instantons*, hep-th/9603167.
- [9] M. Del Zotto, C. Vafa, D. Xie, *Geometric Engineering, Mirror Symmetry and 6d $(1,0) \rightarrow 4d$, $N=2$* , arXiv:1504.08348.
- [10] J. J. Heckman, D. R. Morrison, T. Rudelius, C. Vafa, *Geometry of 6D RG Flows*, arXiv:1505.00009.
- [11] C. Vafa, *Evidence for F-theory*, Nucl. Phys. **B 469** (1996) 403, hep-th/9602022.
- [12] D. R. Morrison and C. Vafa, *Compactifications of F-Theory on Calabi-Yau Threefolds ? II*, Nucl. Phys. **B476** (1996) 437, arXiv:hep-th/9603161.
- [13] W. Fulton, *Introduction to Toric varieties*, Annals of Math. Studies, **No .131**, Princeton University Press, 1993.

- [14] A. Belhaj, *F-theory Duals of M-theory on G_2 Manifolds from Mirror Symmetry*, J. Phys. **A 36** (2003) 4191, hep-th/0207208.
- [15] Y. Aadel, A. Belhaj, Z. Benslimane, M. B. Sedra, A. Segui, *Qubits from Adinkra Graph Theory via Colored Toric Geometry*, arXiv:1506.0252.
- [16] S. Gukov, C. Vafa, E. Witten, *CFT's From Calabi-Yau Four-folds*, Nucl.Phys.**B584**(20009)69, hep-th/9906070
- [17] A. Belhaj, *Manifolds of G_2 Holonomy from $N=4$ Sigma Model*, J. Phys.A35(2002)8903, arXiv:hep-th/0201155.
- [18] R. Ahl Laamara, A. Belhaj, L. J. Boya, L. Medari, A. Segui, *On F-theory Quiver Models and Kac-Moody Algebras*, Int.J.Geom.Meth.Mod.Phys. 7(2010)989, arXiv:0910.4852
- [19] E. Witten, *Phases of $N = 2$ theories in two dimensions*, Nucl. Phys. **B403** (1993), 159, hep-th/9301042.
- [20] M. B. Sedra *Explicit Derivation of New Hyper-Kahler metric*, Nucl.Phys. **B513** (1998) 709, arXiv:hep-th/9707145